Math 24 Discussion Section

Warm Up

With your group, agree upon clear definitions of the following terms:

Function	Differential Operator	Wronskian
Fundamental Set of Solutions	Superposition	

Discuss the following theorem. In particular, what does the phrase "if and only if" mean in this context?

Theorem: Suppose that y_1 and y_2 are two solutions of the initial value problem $y'' + p(t)y' + q(t)y = 0$, $y(t_0) = y_0$, and $y'(t_0) = y_0'$. Then it is always possible to choose constants c_1 and c_2 such that $y = c_1 y_1(t) + c_2 y_2(t)$ is a solution if and only if the Wronskian $W[y_1, y_2]$ is non-zero at t_0.

Problems

1. Compute the Wronskian $W[t^2 e^{-3t}, \cos^2(t)]$.
2. Verify that $y_1(x) = x$ and $y_2(x) = xe^x$ are solutions to $x^2 y'' - x(x+2)y' + (x+2)y = 0,\ x > 0$. Do they form a fundamental set of solutions?
3. A complex number is some element $z \in \mathbb{C}$ which may be expressed as $z = a + bi$ where the real part, $\Re(z) = a$, and the imaginary part, $\Im(z) = b$, are both real numbers and where the imaginary unit i satisfies $i^2 = -1$.

   If $z = a + bi$ is some complex number, then it has complex conjugate $\overline{z} = a - bi$ and modulus $|z| = \sqrt{a^2 + b^2}$.

   (a) For $z = a + bi$, compute $\overline{\overline{z}}$ and $z\overline{z}$.

   (b) On the complex plane, graph and label points $z_1 = -3 + i$, $z_2 = \overline{z_1}$, and $z_3 = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} i$.

   Euler's formula tells us that for any real number $x \in \mathbb{R}$, $e^{ix} = \cos(x) + i\sin(x)$.

   (c) Using Euler's formula, graph and label points $z_4 = e^{i(\pi/2)}$, $z_5 = e^{i(-\pi/3)}$, $z_6 = e^{i(\pi/3)}$, and $z_7 = 3e^{i(\pi/4)}$.

   (d) Compute $|z_1|$, $|z_3|$, $|z_5|$ and $|z_7|$.
4. Find the general solution of $6y'' + 6y' + 3y = 0$.
5. Find the solution to $y'' + 2y' + 2y = 0,\ y(\pi/4) = 2,\ y'(\pi/4) = -2$.
6. Use Euler's formular to show that

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}, \quad \text{and} \quad \sin(x) = \frac{e^{ix} - e^{-ix}}{2i}.$$