Math 24 Discussion Section

Warm Up

With your group, agree upon clear definitions of the following terms:

Logistic Equation | Phase Plot/Line | Stable/Unstable/Semistable Equlibrium

Try to describe the following theorem in your own words and in "plain language." Drawing a picture may help.

Theorem 2.4.2 Let the functions f and $\partial f/\partial y$ be continuous in some rectangle $\alpha < t < \beta$, $\gamma < y < \delta$ containing the point (t_0, y_0). Then, in some interval $t_0 - h < t < t_0 + h$ contained in $\alpha < t < \beta$, there is a unique solution $y = \varphi(t)$ of the initial value problem.

Consider the initial value problem $\dot{x} = x^{2/3}$, $x(0) = 0$. Show that $x(t) = 0$ and $x(t) = \frac{t^3}{27}$ are both valid solutions. Why, exactly, does this observation not contradict the preceding theorem?

Problems

1. Determine an interval in which the solution to the initial value problem $(t^2 - 5)y' + \ln(t)y = t,\ y(3) = 1$ is certain to exist.'

2. Solve the initial value problem $y' + 2y = g(t),\ y(0) = 0$ where $g(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & t > 1 \end{cases}$.

3. For the following autonomous differential equations, find and categorize all equilibria and construct a phase plot.

   (a) $y' = y - 2$ (b) $y' = \dfrac{y-3}{y+9},\ y \ge 0$ (c) $y' = y(3-y)(25-y^2)$.

4. Suppose that the population dynamics of a species obeys a modified version of the logistic differential equation having the following form:

$$\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right)^2 N$$

   where $r \neq 0$ and $K > 0$.

   (a) Show that $\hat{N} = 0$ and $\hat{N} = K$ are equilibria.
   (b) For which values of r is the equlibrium $\hat{N} = 0$ unstable?
   (c) Apply the local stability criterion to the equilibrium $\hat{N} = K$.
   (d) Construct two phase plots, one for the case where $r > 0$ and othe the other for $r < 0$, and determine the stability of $\hat{N} = K$ in each case.