Math 24 Discussion Section

Warm Up

With your group, answer the following questions.

1. What are the requirements to apply the method of integrating factors?
2. What are the steps to applying the method of integrating factors?
3. What makes an ordinary differential equation separable?
4. What are the steps to solving a separable differential equation?
5. Broadly, what makes these two techniques work?

Problems

1. Consider the equation $y' - 2y = t^2 e^{2t}$.
   a. Draw the related direction field. What behavior do you expect as $t \to \infty$?
   b. Find the general solution to the equation, and describe solution behavior as t grows large.
2. Find the general solution for the following ordinary differential equations.
   a. $\frac{dy}{dt} - 2y = 3e^t$ b. $\dot{y} + y = 5\sin(2t)$ c. $\frac{dy}{dx} = \frac{x^2}{1+y^2}$
3. Find the solution to the following initial value problems.
   a. $\frac{dr}{d\theta} = \frac{r^2}{\theta}, \quad r(1) = 2.$
   b. $ty' + (t+1)y = t, \quad y(\ln(2)) = 1, \quad t > 0.$
4. Solve the following initial value problem and determine any extrema of the solution.

$$y' = 2y^2 + xy^2, \quad y(0) = 1$$

5. Find the solution to the following initial value problem.

$$\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1, \quad y = \left(\frac{\pi}{4}\right) = 3\sqrt{2}, \quad 0 < x < \frac{\pi}{2}$$

6. Construct a first-order linear differential equation such that all solutions approach the curve $y = 4 - t^2$ as $t \to \infty$.
   Hint: Consider $y(t) = 4 - t^2 + \eta(t)$ where $\eta(t)$ is some function which decays more rapidly than $4 - t^2$ grows.