Math 24 Discussion Section

Concept Review

Recall the formulas described by or associated with the following. Where appropriate, consider the case for systems of ODEs.

- Euler's Formula
- Variation of Parameters
- Abel's Theorem
- Reduction of Order
- Principle of Superposition
- Abel's Formula

Undetermined Coefficients

Consider the equation $ay'' + by' + cy = g(t)$.

1. What forms can the complementary solution take?
2. Supposing $g(t)$ is the exponential function, the sine or cosine function, a polynomial, or some combination thereof. What are reasonable guesses for the particular solution?

Stability

Consider a system $x' = Ax$ with A a 2×2 matrix. Suppose $\det(A) \neq 0$ and $\lambda_n = a_n + b_n i$ are the eigenvalues of A. If v_n are the associated eigenvectors, we expect a general solution

$$x = c_1 v_1 e^{\lambda_1 t} + c_2 v_2 e^{\lambda_2 t} \quad \text{or} \quad x = c_1 v_1 e^{\lambda_1 t} + c_2 (t v_1 + w) e^{\lambda_1 t}.$$

Discuss the long term stability of the system when

- $a_1 > a_2 > 0,\ b_1 = b_2 = 0$
- $a_1 < 0 < a_2,\ b_1 = b_2 = 0$
- $a_1 = a_2 < 0,\ b_1 = b_2 = 0$
- $a_1 < a_2 < 0,\ b_1 = b_2 = 0$
- $a_1 = a_2 > 0,\ b_1 = b_2 = 0$
- $a_1 = a_2 < 0,\ b_1 = -b_2 \neq 0$
- $a_1 = a_2 > 0,\ b_1 = -b_2 \neq 0$
- $a_1 = a_2 = 0,\ b_1 = -b_2 \neq 0$

Practice

1. Find a solution for $y'' - 2y' + y = te^t + 4,\ y(0) = 1,\ y'(0) = 1$.
2. Find the general solution to $y'' + 9y = 9\sec^2(3t),\ 0 < t < \pi/6$.
3. Consider $x' = Px = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} x;\ x^{(1)} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t},\ x^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$.
   (a) Show that $x^{(1)}$ and $x^{(2)}$ form a fundamental set of solutions.
   (b) Show that $x = c_1 x^{(1)} + c_2 x^{(2)}$ is a solution for any choice of constants.
   (c) Find a solution to the system given the initial condition $x(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
   (d) Compute the Wronskian.
   (e) Show that the Wronskian solves $W' = (p_{11}(t) + p_{22}(t))W$.
4. Solve $x' = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix} x,\ x(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, and describe the behavior of the solution as $t \to \infty$.