Math 145 Discussion Section

Warm Up

Concept Review

With your group, discuss the following terms from lecture. Try to come up with both a clear, plain language definition and a rigorous, mathematical definition with appropriate formulas and expressions.

Diameter (of a Set) Hausdorff Pre-Measure Hausdorff Measure
Hausdorff Dimension

Interlude:

Problems

1. Let $S \subseteq \mathbb{R}^2$ be the unit square and $\mathcal{H}^s(S)$ the s-dimensional Hausdorff measure
   (a) Compute $\mathcal{H}^s(S)$ for $s > 2$.
   (b) Compute $\mathcal{H}^s(S)$ for $s < 2$.
   (c) Conclude that the Hausdorff dimension, at least in this case, agrees with intuition.
2. (a) Show the scaling property of the Hausdorff measure, $\mathcal{H}^s(\lambda A) = \lambda^s \mathcal{H}^s(A)$.

   A similarity of ratio r is a map $T(x) = rAx + t$ such that A is an orthogonal matrix and t is a translation. A set E is self-similar if it may be written as the union of essentially disjoint sets $T_i(E)$ with T_i similarities.

   (b) Show that if $E = \bigcup_{i=1}^{N} T_i(E)$ with $T_i(E)$ pairwise disjoint similarities of ratio r, then we have that the Hausdorff dimension $s = \frac{\log(N)}{\log(1/r)}$.
3. Beginning with an equilateral triangle:
   i. Subdivide the triangle into four, smaller, congruent triangles.
   ii. Remove the middle triangle.
   iii. Repeat this subdivision and removal process on all unremoved sub-triangles.

   The resultant set is called the Sierpinski Triangle.

   (a) Draw the first few iterations of the Sierpinski Triangle.
   (b) Compute the Hausdorff dimension of the Sierpinski Triangle.