

Math 145 Discussion Section

Warm Up

Concept Review

With your group, discuss the following terms from lecture. Try to come up with both a clear, plain language definition and a rigorous, mathematical definition with appropriate formulas and expressions.

Closure of a Set

Fractal

Totally Disconnected Set

Perfect Set

The Metric Space (Σ_2, d)

C^r -Structural Stability

Fundamental Domain

Isomorphism

Homeomorphism

Diffeomorphism

Interlude: Sequence Spaces and Function Spaces

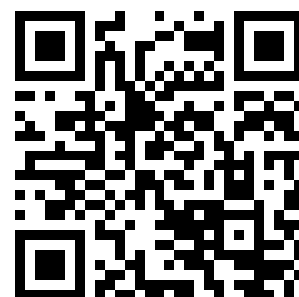
Problems

- Let $s = (0010\ 0010\ 0010\ \dots)$, $t = (000\ \dots)$, and $r = (01\ 01\ 01\ \dots)$ in (Σ_2, d) . Find
 - $d[s, t]$.
 - $d[s, r]$.
- Consider the logistic map F_5 along with its associated Cantor set Λ and itinerary map $S : \Lambda \rightarrow \Sigma_2$. Find the $x \in [0, 1]$ whose itinerary is
 - $s = (1110000\ \dots)$.
 - $s = (101000\ \dots)$.
- Determine if

$$d_r(f, g) := \sup_{x \in \mathbb{R}} \{|f(x) - g(x)|, |f'(x) - g'(x)|, \dots, |f^{(r)}(x) - g^{(r)}(x)|\}$$

defines a distance on $C^r(\mathbb{R})$.

- (Devaney, ex. 2 p. 72) Let $T_{-1}(x) = x^3 + x$. Prove that T_{-1} is not structurally stable.
 - Determine any fixed points and whether they are hyperbolic.
 - Determine the same for a small perturbation $T_{-1 \pm \varepsilon}(x) = x^3 + (1 \mp \varepsilon)x$.
 - Are T_{-1} and $T_{-1 \pm \varepsilon}$ C^1 close on \mathbb{R} ? On the interval $[-2, 2]$?
 - Conclude that T_{-1} is not structurally stable locally.



Extra Problems

1. Consider Arnold's cat map $f : \mathbb{T}^2 \rightarrow \mathbb{T}^2$ given by $f(x, y) = (2x + y, x + y) \pmod{1}$
 - (a) Determine any fixed points of the map f .
 - (b) Compute the Jacobian and its eigenvalues.
 - (c) Conclude that all fixed points are hyperbolic, and therefore the map is C^1 -structurally stable locally.