Math 145 Discussion Section

Warm Up

Concept Review

With your group, discuss the following terms from lecture. Try to come up with both a clear, plain language definition and a rigorous, mathematical definition with appropriate formulas and expressions.

Closure of a Set	Fractal	Totally Disconnected Set
Perfect Set	The Metric Space (Σ_2, d)	C^r-Structural Stability
Fundamental Domain		
Isomorphism	Homeomorphism	Diffeomorphism

Interlude: Sequence Spaces and Function Spaces

Problems

1. Let $s = (0010\ 0010\ 0010\ \cdots)$, $t = (000\ \cdots)$, and $r = (01\ 01\ 01\ \cdots)$ in (Σ_2, d). Find
   (a) $d[s, t]$.
   (b) $d[s, r]$.
2. Consider the logistic map F_5 along with its associated Cantor set Λ and itinerary map $S : \Lambda \to \Sigma_2$. Find the $x \in [0, 1]$ whose itinerary is
   (a) $s = (1110000\cdots)$.
   (b) $s = (101000\cdots)$.
3. Determine if
$$d_r(f, g) := \sup_{x\in\mathbb{R}}\{|f(x) - g(x)|,\ |f'(x) - g'(x)|, \ldots, |f^{(r)}(x) - g^{(r)}(x)|\}$$
   defines a distance on $C^r(\mathbb{R})$.
4. (Devaney, ex. 2 p. 72) Let $T_{-1}(x) = x^3 + x$. Prove that T_{-1} is not structurally stable.
   (a) Determine any fixed points and whether they are hyperbolic.
   (b) Determine the same for a small perturbation $T_{-1\pm\varepsilon}(x) = x^3 + (1 \mp \varepsilon)x$.
   (c) Are T_{-1} and $T_{-1\pm\varepsilon}$ C^1 close on $\mathbb{R}$? On the interval $[-2, 2]$?
   (d) Conclude that T_{-1} is not structurally stable locally.

Extra Problems

1. Consider Arnold's cat map $f : \mathbb{T}^2 \to \mathbb{T}^2$ given by $f(x, y) = (2x + y, x + y) \pmod 1$
   (a) Determine any fixed points of the map f.
   (b) Compute the Jacobian and its eigenvalues.
   (c) Conclude that all fixed points are hyperbolic, and therefore the map is C^1-structurally stable locally.