Math 145 Discussion Section

Warm Up

Concept Review

With your group, discuss the following terms from lecture. Try to come up with both a clear, plain language definition and a rigorous, mathematical definition with appropriate formulas and expressions.

Open Set (in $\mathbb{R}$)	Distance Function	Open Set (In a Metric Space)
Neighborhood of a Point	Chaotic Map	Logistic Map
Topological (Semi-)Conjugacy	Compact Set	Multiplier (of a Fixed Point)
Bifurcation		

Interlude: Questions and a Brief Discussion of Topology

Problems

1. (Devaney, ex. 1 p. 38) Find all periodic points for each of the following maps and classify them as attracting, repelling, or neither. [Compute the multipliers of fixed points,] sketch the [cobweb diagrams and] phase portraits.

   (a) $f(x) = x^3 - x$ (g) $E(x) = e^{x-1}$

2. (a) Show that if $S \subseteq \mathbb{R}$ is finite then it admits no limit points.

   (b) Does this hold for $S \subseteq X$ where X is an arbitrary metric space (X, d_X)?

3. Consider a real sequence $\{u_n\}_{n=1}^{\infty}$ and its associated set $\{u_n \ : \ n \in \mathbb{Z}^+\}$. Determine the limit sets of both when

   (a) $u_n = \dfrac{1}{n}$ (b) $u_n = (-1)^n$

4. Let $F_\mu(x) = \mu x(1-x)$, $\mu > 0$, denote the logistic map and $Q_c(x) = x^2 + c$, $c \in \mathbb{R}$, denote the quadratic map.

   (a) Show that the map $g(x) = -4x + 2$ conjugates Q_{-2} and F_4.

   (b) More generally, show that there exists a map $h(x) = ax + b$ which conjugates Q_c and F_μ for some choice of $a, b, c \in \mathbb{R}$. Determine $a, b, c \in \mathbb{R}$ in terms of μ.

5. (Devaney, ex. 2 p. 38) Discuss the bifurcations which occur in the following families of maps for the indicated parameter value.

   (b) $E_\lambda(x) = \lambda e^x, \quad 1/e$ (d) $Q_c(x) = x^2 + c, \quad c = -3/4$

Extra Problems

2. (c) Describe a space X and a finite subset $S \subseteq X$ which admits a limit point.