Math 145 Discussion Section

Warm Up

Concept Review

With your group, discuss the following terms from lecture. Try to come up with both a clear, plain language definition and, if possible, a rigorous definition.

Dynamical System
n-Periodic
Cobweb Diagram
Stable/Unstable Sets
Limit Set of a Set/Sequence

Orbit
Prime Period
Convergence (of a Sequence)
Limit Point of a Set
Dense Set

Stable/Unstable Fixed Point
n-Cycle
Continuous Function
Limit Point of a Sequence

Notation Review

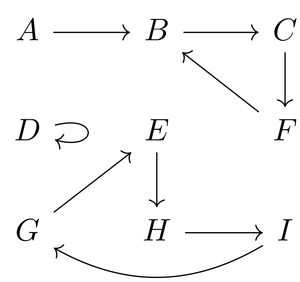
With your group, translate these sentences into English and identify the terms above to which they correspond.

1. For $S \subset \mathbb{R}$, $\forall \varepsilon > 0,\ \exists x \in S,\ x \neq y$ and $|x - y| < \varepsilon$.
2. For $\{x_n\}_{n \geq 0}$ real-valued, $\forall \varepsilon > 0,\ \forall N \in \mathbb{N},\ \exists n \geq N$ such that $|x_n - y| < \varepsilon$.

Interlude - Mathematical Proofs

Problems

1. Suppose the directed graph describes a discrete dynamical system.
   (a) Draw f^2 and f^3.
   (b) Describe the orbit $\mathcal{O}^+(x)$ for each $x \in S$.
   (c) Describe $\text{Per}_k(f)$ for $k = 1, 2, 3, 4$.
   (d) Identify any cycles and their periods.
   (e) Which points are *eventually periodic*?



2. Consider the function $x^2 - 1$. Identify, by cobwebbing, any fixed points and cycles.
3. Fixing $\mu > 0$, consider the map $F_\mu(x) = \mu x(1 - x)$:
   (a) Show that for $0 < \mu \leq 4$, we have that $F_\mu([0, 1]) \subset [0, 1]$, so that we can study the dynamics of the box.
   (b) For $\mu = 1/2$, discuss the fixed points of f and their stable set. Can you predict all orbit behaviors from the initial conditions in $[0, 1]$ by cobwebbing?
   (c) Repeat the previous question with $\mu = 2$.
4. (Devaney, ex. 7 p. 31) Prove that a homeomorphism of $\mathbb{R}$ can have no periodic points with prime period greater than 2. Give an example of a homeomorphism that has a periodic point of order 2.