Teaching Assitant

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Office hours:

Mondays 11am-12pm - ARC 116
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Problem 1

Discuss the solutions of Quiz 1 as needed.

Problem 2

Consider the equation $\dot{x} = x^2 + h(x)$ with initial conditions $x(0) = x_0$ and h some function satisfying $h(x) \geq 0$ for all x. Show that for any $x_0 > 0$, $x(t) \to \infty$ in finite time.

Trick: observe that $\dot{x} \geq x^2$ and integrate this.

Problems 1 and 2

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We know that $x(t) = \frac{x_0}{1 - t x_0}$ (compute this yourself).

Problem 3

Consider the $n \times n$ system of linear ODEs $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$ for $A \in \mathbb{R}^{n\times n}$. Suppose that $(\lambda, v) \in \mathbb{C} \times \mathbb{C}^{n\times n}$ is an eigenpair of A (in the sense that $A\mathbf{v} = \lambda\mathbf{v}$). Then show that the function $\mathbf{x}(t) = e^{\lambda t}\mathbf{v}$ solves the ODE.

Problem 4

Consider the equation $\frac{d\mathbf{y}}{dt} = A\mathbf{y}$ where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. It can be show that $e^{tA} = e^t \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$.

(a) Write the true transfer matrix B such that $\mathbf{y}(t + \Delta t) = B\mathbf{y}(t)$. You may need the property that $\begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$ for all reals a, b.

(b) Write the approximate transfer matrix corresponding to the Forward Euler scheme, i.e. the matrix $\hat{B}$ such that $\mathbf{u}_{n+1} = \hat{B}\mathbf{u}_n$, if $\mathbf{u}_n$ is meant to approxiamte $\mathbf{y}(n\Delta t)$.

(c) Repeat the previous question with the Heun scheme.

The goal of this problem is to test the Forward Euler scheme on the initial value probem $\dot{x} = x,\ x(0) = 1$.

(a) Solve the problem analytically. What is the exact value of $x(1)$?

(b) Estimate $x(1)$ numerically (Call it $\hat{x}(1)$) using the Forward Euler scheme with step size $\Delta t = 10^{-n}$ for $n = 0, 1, 2, 3, 4$.

(c) Plot the error $E = |\hat{x}(1) - x(1)|$ as a function of Δt. Then plot E versus $\ln(t)$. Explain the result (does this corroborate the convergence rate of the form $E \leq C\Delta t$?).

Attendance