Teaching Assitant

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Office hours:

Mondays 11am-12pm - ARC 116
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Problems 1 and 2

Refresher on complex numbers and polar representation $z = \rho e^{i\theta}$, and notice that $z^n = \rho^n e^{in\theta}$.

Consider $z = a + bi \in \mathbb{C}$.

What is the modulus of z (i.e. $|z|$)?

What is the (principal) argument of z (i.e. $\operatorname{Arg}(z)$)?

Write $z_0 = 2 - 2\sqrt{3}i$ in polar form.

Solving $z^n = w$ using polar representation. Example: compute the 4-th roots of $w = 1$, the 4-th roots of $w = -1$, and the cube roots of $w = i$.

We define the (principal) complex logarithm by

$$\operatorname{Log}(z) = \ln(|z|) + i \operatorname{Arg}(z) = \ln(\rho) + i\theta.$$

Problem 3

Describe in complex notation:

(a) The open disk of center z_0 and radius $r < 0$.

(b) The "ray" (half line) coming out of 0 in the direction $(\cos(\theta), \sin(\theta))$.

(c) The right-half plane.

By Euler's formula, if $x \in \mathbb{R}$ then

$$e^{ix} = \cos x + i \sin x.$$

By set comprehension, we write

$$\{x \in X : P(x)\}$$

to mean the collection of all elements x in the set X which satisfy the formula $P(x)$.

Problem 4

Study the map $H(z) = z + \frac{1}{z}$. Maps rays to hyperbolas. Draw them. Restriction of H to the unit circle?

Problem 5

Reverse iterations of the Newton map of $P(z) = z^3 - 1$: how to describe the preimages of w by $N(z) = z - \frac{P(z)}{P'(z)}$.

Problem 6

(an example of topological conjugacy) Fix $a, b \in \mathbb{C}$ with a, b distinct, and consider the Newton map $N(z) = z - \frac{P(z)}{P'(z)}$ of the quadratic polynomial $P(z) = (z-a)(z-b)$. Show that the map $H(z) = \frac{z-a}{z-b}$ induces a topological conjugacy between $N(z)$ and the doubling map $Q_0(z) = z^2$.

Attendance