Chaos Theory (Math 145)

Teaching Assitant

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Office hours:

Mondays 11am-12pm - ARC 116
Thursdays 2:30-3:30pm - ARC 116

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Let (X, d) be a metric space.

(a) True/False? If a set $S \subseteq X$ is not open, then it is closed?

(b) True/False? if a set $S \subseteq X$ is not closed, then it is open?

(c) Give an example of a set $S \subseteq \mathbb{R}$ that is

- **i.** Both open and closed.
- **ii.** Neither open nor closed.
- **iii.** Closed and not open.
- **iv.** Open and not closed.

(d) Show that $\mathbb{Z}$ has no limit point in $\mathbb{R}$.

Problem 1

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Definition: Limit Point of a (Real) Set

We say that y is a limit point of the set $S \subseteq \mathbb{R}$ if

$$\forall \varepsilon > 0,\ \exists x \in S \text{ such that } x \neq y \text{ and } \operatorname{dist}(x, y) < \varepsilon.$$

- What is the *precise* negation of this definition?
- How do we show that an arbitrary point is *not* a limit point of $\mathbb{Z}$?

Problem 2

Let $s = (0010\,0010\,0010\cdots)$, $t = (000\cdots)$, and $r = (01\,01\,01\cdots)$. Find

(a) $d[s, t]$.

(b) $d[s, r]$.

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Definition: $d : \Sigma_2 \times \Sigma_2 \to [0,\infty)$

$$d[s,t] := \sum_{k=0}^{\infty} \frac{|s_k - t_k|}{2^k}$$

Problem 3

(a) Revisit the proximity theorem.

(b) Show that the set of eventually 2-periodic points of σ, i.e.

$$B = \{s \in \Sigma_2 \,:\, \exists n \geq 0,\, \sigma^n(s) \in \mathrm{Per}_2(\sigma)\},$$

is dense in Σ_2.

Problem 3

(a) Revisit the proximity theorem.

Theorem: Proximity Theorem

Let $s, t \in \Sigma_2$.

(1) If $s_k = t_k$ for all $0 \le k \le n$, then $d[s,t] \le \frac{1}{2^n}$.

(2) If $d[s,t] < \frac{1}{2^n}$, then $s_k = t_k$ for all $0 \le k \le n$.

- How can we split the sum $\sum_{k=0}^{\infty} \frac{|s_k - t_k|}{2^k}$ with regards to n?
- What is the contrapositive of **(1)**?

(b) Show that the set of eventually 2-periodic points of σ, i.e.

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is dense in Σ_2.

Definition: Shift Operator

$$\sigma : \Sigma_2 \to \Sigma_2, \quad \sigma((s_0 s_1 s_2 \cdots)) = (s_1 s_2 s_3 \cdots)$$

Consider the map $F_\mu(x) = \mu x(1-x)$ with $\mu = 5$, along with its Cantor set Λ and itinerary map $S : \Lambda \to \Sigma_2$. Find the $x \in [0,1]$ whose itinerary is

(a) $s = (1110000\cdots)$

(b) $s = (1010000\cdots)$

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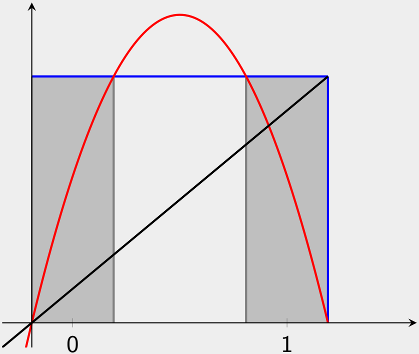

Problem 5

Refresher on vocab: if any notion needs clarification, redefinition, examples, please ask!

(a) topological vocab

(b) dynamical vocab

(c) calculus vocab

Problem 6

Tips about proof-writing and approaching questions with proofs.

Problem 7

Spend last 20 mins on the first 3 HWs as needed.

Attendance