Teaching Assitant

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Office hours:

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Mathematical Vocabulary

Analysis

Field
Archimedean Property
Monotone Convergence
Triangle Inequality
Metric Space
Limits
Continuity
C^n Functions
Critical Points
Partition

Dynamics

Chaos
(Discrete/Continuous) Dynamics
Orbit
Fixed Point
(Prime) Period
Cycle
Eventually Periodic
Cobwebbing
Asymptotic Behavior
Stable Set
Topological Transitivity
Sensitivity to Initial Conditions
(Semi-)conjugacy
Multiplier
Structural Stability
Logistic Family
Chebyshev Polynomials (1st Kind)
Bifrucation
Attracting Cycle

Topology

(Partial/Total) Ordering
Subset
Continuous Function
Limit Point
Density
Open Sets (Neighborhood)

Tips about proof-writing and approaching.

Problem 1

Let J be an interval. Show that if $f\colon J \to J$ is a strictly increasing bijection, then for every a, b such that $(a, b) \subset J$, $f^{-1}((a, b)) = (f^{-1}(a), f^{-1}(b))$.

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- Can a bijection $f : J \to J$ be non-strictly increasing? Non-monotonic?
- What does the graph of f^{-1} look like?

Problem 2

Consider the function $f(x) = x^2 + 2x + 2$. Show that for any nonempty open interval (a, b) with $-\infty \leq a < b \leq \infty$, $f^{-1}((a, b))$ is open (first partition the real line into intervals where f is one-to-one and onto). This will provde that f is continuous on $\mathbb{R}$.

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- What are the critical points of f?
- What is our partition?
- What does the graph of f^{-1} look like?

Fix I, J two intervals. Suppose that $f\colon I \to I$ and $g\colon J \to J$ are semi-conjugate via a C^1-map $h\colon I \to J$, i.e. $h \circ f = g \circ h$.

(a) Show that if p is a fixed point of f, then $h(p)$ is a fixed point of g, and both have the same multiplier.

(b) Can you generalize this to 2-cycles and up?

In order words, topological semi-conjugacies also propagate stable dynamics.

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- What does it mean to be a fixed point of f?
- What is the multiplier of $h(f(p))$, for p a fixed point?

Can you generalize this to 2-cycles and up?

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- Observe that the fixed points of f^n are the n-cycles of f.
- Then the multiplier of the cycles must be the multiplier of f^n.

Attendance