Teaching Assitant

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Office hours:

Mondays 11am-12pm - ARC 116
Thursdays 2:30-3:30pm - ARC 116

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Problem 1

Go over the details of HW1 Q1 with different maps.

$$\begin{tikzcd}
A \arrow[r] & B \arrow[r] & C \arrow[d] \\
D \arrow[loop right] & E \arrow[d] & F \arrow[ul] \\
G \arrow[ur] & H \arrow[r] & I \arrow[ll, bend left]
\end{tikzcd}$$

Exercise

(a) Draw f^2 and f^3.

(b) Describe the orbit $\mathcal{O}^+(x)$ for each $x \in S$.

(c) Describe $\mathrm{Per}_k(f)$ for $k = 1, 2, 3, 4$.

(d) Identify any cycles and their periods.

(e) Which points are *eventually periodic*?

Problem 1 (a)

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\end{tikzcd}$$

Draw f^2 and f^3.

$$\begin{tikzcd}
A \arrow[rr, bend left] & B \arrow[dr] & C \arrow[l] \\
D \arrow[loop right] & E \arrow[dr, bend right] & F \arrow[u] \\
G \arrow[r, bend right] & H \arrow[l, bend right] & I \arrow[ul, bend right]
\end{tikzcd}
\qquad
\begin{tikzcd}
A \arrow[drr] & B \arrow[loop right] & C \arrow[loop right] \\
D \arrow[loop right] & E \arrow[dl] & F \arrow[loop right] \\
G \arrow[rr, bend right] & H \arrow[u] & I \arrow[l]
\end{tikzcd}$$

Problem 1 (b), (c)

Go over the details of HW1 Q1 with different maps.

$$\begin{tikzcd}
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D \arrow[loop right] & E \arrow[d] & F \arrow[ul] \\
G \arrow[ur] & H \arrow[r] & I \arrow[ll, bend left]
\end{tikzcd}$$

Describe the orbit $\mathcal{O}^+(x)$ for each $x \in S$.

e.g. $\mathcal{O}^+(A) = A, B, C, F, B, C, F, \ldots$.

Describe $\mathrm{Per}_k(f)$ for $k = 1, 2, 3, 4$.

e.g. $\mathrm{Per}_3(f) = \{B, C, D, F\}$.

Problem 1 (d), (e)

Go over the details of HW1 Q1 with different maps.

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D \arrow[loop right] & E \arrow[d] & F \arrow[ul] \\
G \arrow[ur] & H \arrow[r] & I \arrow[ll, bend left]
\end{tikzcd}$$

Identify any cycles and their periods.

e.g. the cycle E, G, I, H has period four.

Which points are *eventually periodic*?

e.g. A is eventually periodic; we exclude periodic points from our definition of "eventually periodic."

Problem 2

Go over cobwebbing. Remind yourself why, as long as you are iterating a *function*, forward cobwebbing is always well-defined.

Exercise

Consider the function $x^2 - 1$. Identify, by cobwebbing, any fixed points and cycles.

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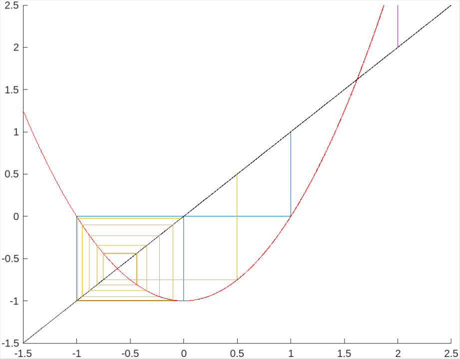

Problem 3

Fixing $\mu > 0$, consider the map $F_\mu(x) = \mu x(1-x)$:

(a) Show that for $0 < \mu \leq 4$, we have that $F_\mu([0,1]) \subset [0,1]$, so that we can study the dynamics of the box.

(b) For $\mu = 1/2$, discuss the fixed points of f and their stable set. Can you predict all orbit behaviors from the initial conditions in $[0,1]$ by cobwebbing?

(c) Repeat the previous question with $\mu = 2$.

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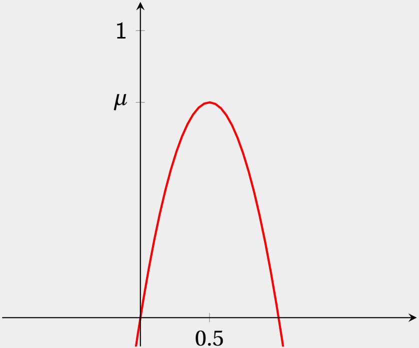

Problem 3 (b)

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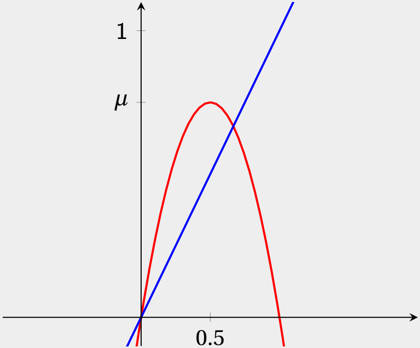

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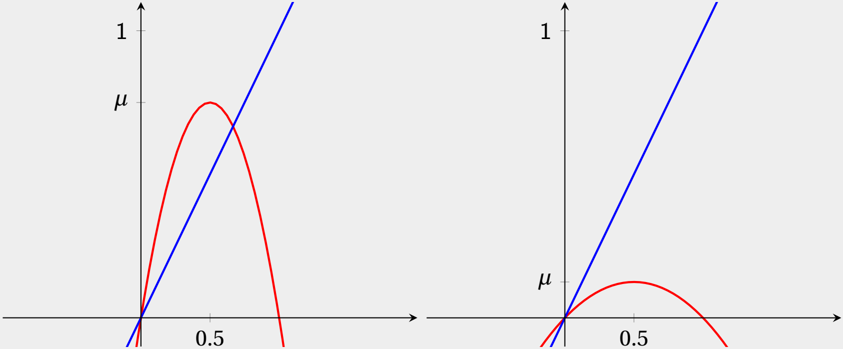

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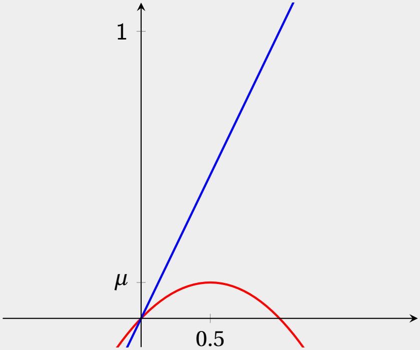

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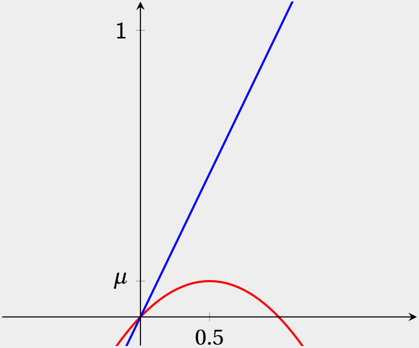

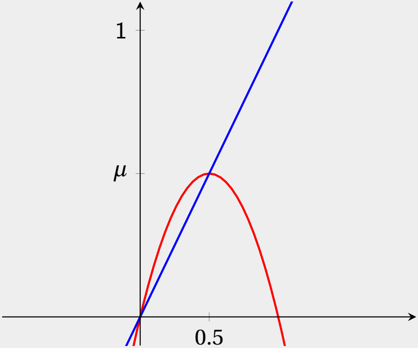

Problem 4

Recall that the map $f(x) = x^2 - 1$ has two repelling fixed points $x_\pm = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$, and a 2-cycle $(0, -1)$ to which many points of the interval $(-x_+, x_+)$ are asymptotic. Some which are not occur for those points that eventually land into x_- (namely, the stable set of x_-, denoted $W^s(x_-)$). In this context, this set is constructed running dynamics backwards on x_-, specifically

$$W^s(x_-) = \bigcup_{n \geq 0} f^{-n}(x_-), \quad \text{where} \quad f^{-n}(x_-) := \{y : f^n(y) = x_-\}.$$

Note that since f is not one-to-one (often 2-to-1 here), or invertible, $f^{-n}(x_-)$ is a set which contains potentially in the order of 2^n points! *luckily, it does not grow that fast!)

Discuss how to construct this set by "reverse"-cobwebbing, describing some qualitative features of it and thinking about an algorithmic way to construct it, as this will help you with a Matlab problem.

Attendance