## Math 11B Discussion Section

With your group, come up with plain language definitions for the following terms:

- Integration
- Order (of a differential equation)
- Initial Condition
- Equilibrium
- Differential Equation
- Solution (of a differential equation)
- Phase Plot
- Local Stability

(1) Show that $y = \frac{2}{3}e^x + e^{-2x}$ is a solution to the differential equation $y' + 2y = 2e^x$. Is this differential equation pure-time, autonomous, or nonautonomous?

(2) The Weibull equation for the dynamics of the drug concentration is

$$\frac{dc}{dt} = \frac{k}{t^b}(c_s - c)$$

where $k$, $c_s$, and $b$ are positive constants and $b < 1$. Notice that this differential equation is undefined when $t = 0$. Is this differential equation pure-time, autonomous, or nonautonomous? State in words what this differential equation says about how drug dissolution occurs. Verify that

$$c = c_s\left(1 - e^{\alpha t^{1-b}}\right)$$

is a solution for $t \neq 0$, where $\alpha = k/(1-b)$.

(3) Find all equilibria of the autonomous differential equation and construct the phase plot

(a) $y' = y^2 - 2$

(b) $y' = \frac{y-3}{y+9}, \quad y \geq 0$

(c) $y' = y(3-y)(25-y^2)$

(4) Suppose that the population dynamics of a species obeys a modified version of the logistic differential equation having the following form:

$$\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right)^2 N$$

where $r \neq 0$ and $K > 0$.

(a) Show that $\hat{N} = 0$ and $\hat{N} = K$ are equilibria.

(b) For which values of $r$ is the equlibrium $\hat{N} = 0$ unstable?

(c) Apply the local stability criterion to the equilibrium $\hat{N} = K$. What do you think your answer means about the stability of this equilibrium?

(d) Construct two phase plots, one for the case where $r > 0$ and othe other for $r < 0$, and determine the stability of $\hat{N} = K$ in each case.