Math 11B Discussion Section

1. (a) What is a differential equation?
   (b) What is the order of a differential equation?
   (c) What is an initial condition?
   (d) What are the differences between pure-time, autonomous, and nonautonomous differential equations?
2. What is a phase plot for the differential equation $y' = g(y)$?
3. What is a direction field for the differential equation $y' = F(x, y)$?
4. Explain how Euler's method works.
5. What is a separable differential equation? How do you solve it?
6. What is a nullcline?
7. For the differential equation $x' = ax + x^3$,
   (a) Determine all equilibria as a function of the constant a.
   (b) Construct phase plots for (i) $a > 0$, (ii) $a = 0$, and (iii) $a < 0$, and use it to identify stability of any equilibria.
8. For the differential equation $y' = x/y$,
   (a) Sketch a direction field, and sketch the four solutions that satisfy the initial conditions $y(0) = 1$, $y(0) = -1$, $y(2) = 1$, and $y(-2) = 1$.
   (b) Solve the differential equation explicitly. What type of curve is each solution?
9. Solve the differential equation $2ye^{y^2}y' = 2x + 3\sqrt{x}$.
10. Solve the initial value-problem $(1 + \cos x)y' = (1 + e^{-y})\sin x$, $y(0) = 0$.
11. The per capita growth rate of a population varies seasonally and habitat destruction is also occurring. This is modeled as

$$n' = r\left(\cos\left[\frac{2\pi t}{365}\right] - at\right)n, \quad n(0) = n_0$$

    where $n(t)$ is the population size at time t (measured in days) and r and a are positive constants. Determine the population size at time t.

12. The process of cell division is periodic, with repeated growth and division phases as the cell population multiplies. It has been suggested that the division phase is triggered by high concentrations of a molecule called MPF (maturation promoting factor). The production of this factor is stimulated by another molecule called cyclin, and MPF eventually inhibits its own production. Using M and C to denote the concentrations of these two biomolecules (in mg/ml), a simple model for their interaction is

$$\frac{dM}{dt} = \alpha C + \beta C M^2 - \frac{\gamma M}{1+M}, \qquad \frac{dC}{dt} = \delta - M.$$

    (a) Suppose that $\alpha = 2$, $\beta = 1$, $\gamma = 10$, and $\delta = 1$. Construct the phase plane, including all nullclines, equilibria, and arrows indicating the direction of movement in the plane.
    (b) From your answer to part (a), what is the qualitative nature of the dynamics of M predicted by this model? What does this predict about the dynamics of cell division?
    (c) For any equilibrium found in part (a), specify whhether it is locally stable, unstable, or if the information is inconclusive.