Question of the Day

If you had to swap your legs with the legs of any other animal, which animal would you choose?

On the Docket

Check in.
Concept Review – Approximation and Optimization
Function Shapes
Harmonic Constituents?
Chain rule practice.

Approximation

First Order Approximation

$$f(x+dx) \approx f(x) + f'(x) \cdot dx$$

Second Order Approximation

$$f(x+dx) \approx f(x) + f'(x) \cdot dx + \frac{1}{2} f''(x) \cdot (dx)^2$$

- What is f?
- What is $\approx$?
- What is x?
- What is dx?

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To approximate $\sqrt[3]{127}$

- What is f?
- What is f'?
- What is f''?
- What is x?
- What is dx?

Approximation

Question

What might a third order approximation look like?

Exercise

Find a first and second order approximation of $f(x) = \sqrt{x}$.
Use $x = 4$ and $dx = -0.1$ to approximate $\sqrt{3.9}$.
Try $x = 4$ and $dx = 5$ to approximate $\sqrt{9}$.
What is happening?

Exercise

The diameter of a tumor was measured to be 19 mm. If the diameter increases by 1 mm, use a first order approximation to estimate the relative change in volume ($V = \frac{4}{3}\pi r^3$) and surface area ($S = 4\pi r^2$).

Goal and Constraint

An optimization problem consists of a goal and a restriction

$$g(x, y) \qquad\qquad r(x, y) = C$$

Using the restriction, rewrite the goal in terms of one variable.
With the derivative, optimize the goal (e.g. maximize, minimize).

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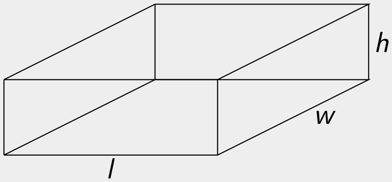
Exercise

If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

- What is the goal function?
- What is the constraint?
- What is the restriction?

Optimization Solution



Restriction: $A(h, l, w) = 2lh + 2wh + wl$.
Goal: $V(h, l, w) = lwh$.
Since the box is square bottomed, $l = w$. Therefore
$A(h, w) = 2wh + 2wh + ww = 4wh + w^2, \quad V(h, w) = wwh = w^2 h$.
Given $4wh + w^2 = 1200$, rearrange to $h = 300w^{-1} - \frac{1}{4}w$. Then

$$V'(w) = \frac{d}{dw} w^2 \left(300w^{-1} - \frac{1}{4}w\right) = \frac{d}{dw} 300w - \frac{1}{4}w^3 = 300 - \frac{3}{4}w^2$$

Since $300 - \frac{3}{4}w^2 = 0$ when $w = \pm 20$, the volume maximizes at

$$300w - \frac{1}{4}w^3 = 300(20) - \frac{1}{4}(20)^3 = 4000$$

Optimization

Exercise

Find two positive number whose product is 100 and whose sum is a minimum.

Exercise

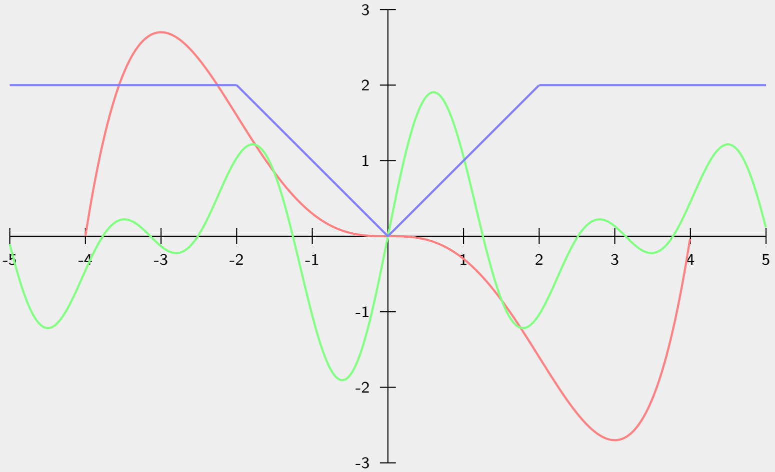
The sum of two positive numbers is 16. What is the smallest possibel value of the sum of their squares?

Question

If a system to be optimized is written in terms of <u>three</u> variables, what information might make the system solvable?

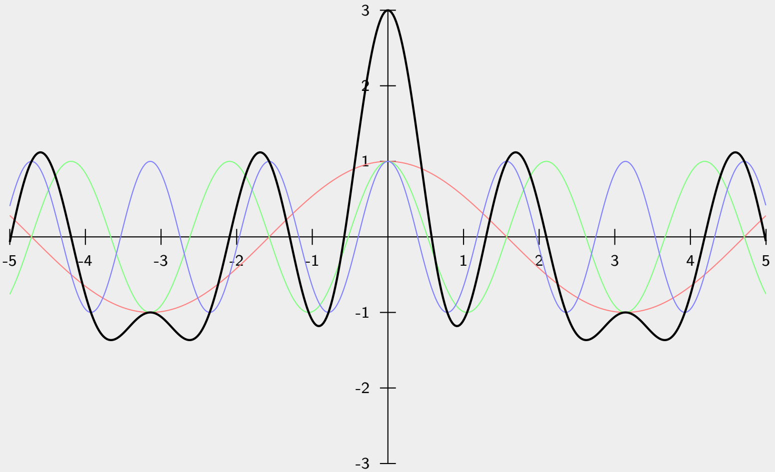
Function Shapes

At the points $x \in \{-3, -2, -1, 0, 1, 2, 3\}$ below

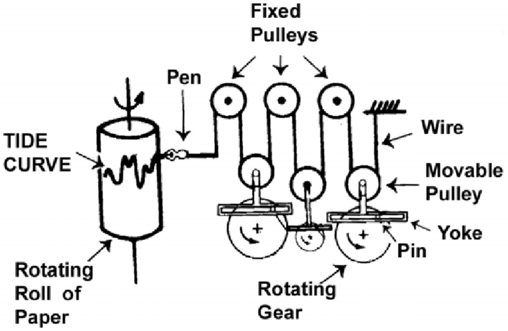


What can you say about the functions red, green and blue?
(Think slope, first derivative, second derivative, etc.)

Harmonic Constituents



Harmonic Constituents



Chain Rule Practice

Differentiate the following functions:

- $f(x) = (6x^2 + 7x)^4$
- $g(t) = (4t^2 - 3t + 2)^{-2}$
- $H(z) = 2^{1-6z}$
- $h(z) = \sin(z^6) + \sin^6(z)$
- $f(x) = \ln(\sin(x)) - (x^4 - 3x)^{10}$
- $f(x) = (\sqrt[3]{12x} + \sin^2(3x))^{-1}$