Question of the Day

What is your favorite thing to eat on Thanksgiving?

On the Docket

Check In
Function Review
Derivative Rule Practice
Approximations

Functions

```
newtonsMethod[function_, steps_, guess_] := (
  Module[{i, n, derivatives = {function, D[function, x]}, approximation = {N[guess]}},
   For[i = 1, i < steps + 1, i ++,
    approximation = AppendTo[approximation, N[x - (derivatives[[1]] / derivatives[[2]]) /. x → approximation[[i]]]];
   ];
   Grid[{
     {TableForm[{
        "Newton's Method",
        {},
        StringForm[" f(x) = ``", function],
        StringForm[" `` = ``", Subscript["p", i - 1], NumberForm[approximation[[i - 1]], 10]],
        StringForm["|``-``| = ``", Subscript["p", i - 1], Subscript["p", i - 2], NumberForm[Abs[approximation[[i - 1]] - approximation[[i - 2]]], 10]]}],
      TableForm[Table[{
         Subscript["p", n],
         approximation[[n + 1]]
        }, {n, 0, steps}]]
     }}
    , Frame → All]
  ]
 )
```

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         approximation[[n + 1]]
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     }}
    , Frame → All]
  ]
 )
```

- Why do we call them functions?
- What *is* a function?

Functions

```
In[1]:=
        addition[x_, y_] := (
          x + y
         )

In[2]:= exponentiation[x_, n_] := (
          x ^ n
         )

        addition[a, exponentiation[z, k]]
```

- What is happening in In[1]? In[2]?
- What about the final line?
- Can you rewrite it in "function notation" (i.e. $f(x) = y$)?
- Can you build a "Pacman" or "black box" diagram for the last line?

Derivative Rules

For the following, first build a diagram, naming the individual functions, then take the derivative with respect to x.

- $\frac{1}{4}x^7 + x^{-1}$
- $2x^{3/2}$
- $\frac{\sin(x)}{3-2\cos(x)}$
- $\frac{5}{2}x^{x+3/2} + \frac{5}{2}a^3$
- $\ln\left(\frac{x+1}{\sqrt{x-2}}\right)$
- $\exp(ax) - \exp(bx^3)$

Approximations

Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Use Newton's Method with the specified initial approximation x_1 to find x_3.

- $x^3 + 2x - 4 - 0, \quad x_1 = 1$
- $\frac{1}{3}x^3 + \frac{1}{2}x^2 + 3 = 0, \quad x_1 = -3$

Taylor Polynomials

The second degree Taylor polynomial of f centered at a is given as

$$P(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$$

- Find the second degree Taylor polynomial $T_2(x)$ centered at $a = 0$ for the function $f(x) = \cos(x)$.
- Find the second degree Taylor polynomial $T_2(x)$ centered at $a = 4$ for the function $f(x) = \sqrt{x}$.
- Find the *fourth* degree Taylor polynomial $T_4(x)$ centered at $a = 1$ for the function $f(x) = \frac{1}{x}$.