Question of the Day

What is your favorite class right now?

On the Docket

Concept check in.
Rule Review
Derivative Practice

Power

Let n be a constant. Then

The Power Rule

$$\frac{d}{dx}x^n = n \cdot x^{n-1}$$

- What is the first derivative of $f(x) = \beta x^{\alpha}$?
- What is "the" derivative of $x^{13}y^7z^3$?
- What about $\frac{d}{dx}13^x$?
  Does the power rule work here?
  Why or why not?

Sum and Multiplication

Suppose n is a constant and f and g are differentiable at x. Then

The Sum Rule

$$\frac{d}{dx}[n \cdot f(x)] = n \cdot \frac{d}{dx} f(x)$$

The Multiplication Rule

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

- The restrictions are different from the power rule. How and why?
- Calculate $\frac{d}{dt}(t^3 - 8t^2 - 3^t)$.

Chain

If g is differentiable at x and f is differentiable at $y = g(x)$, then the composite function $(f \circ g)(x) = f[g(x)]$ is differentiable at x, and the derivative is given by

The Chain Rule

$$(f \circ g)'(x) = f'[g(x)] \cdot g'(x)$$

- What does $(f \circ g)$ mean?
  How do we read it?
- Find the derivative of $h(x) = (3x^2 - 1)^2$.
- Calculate $\frac{df}{du}\frac{du}{dx}\sqrt{x^2 + 1}$.

Interlude: The Binomial Theorem

Pascal's Triangle

					1							
			1		2		1					
		1		3		3		1				
	1		4		6		4		1			
1												

- What is happening here?
- Can you continue the triangle (at least two tiers)?
- Expand $(x + y)^6$.

Problem 1

A car moves along a straight road. Its location at time t is given by

$$s(t) = 3t - t^2,\ 0 \leq \text{t} \leq 3$$

where t is measured in hours and s(t) is measured in kilometers.
(a) Graph s(t) for $0 \leq \text{t} \leq 3$.
(b) Find the average velocity of the car between $\text{t} = 1$ and $\text{t} = 2$. Illustrate the average velocity on the graph of s(t).
(c) Use calculus to find the instantaneous velocity of the car at $\text{t} = 1.5$. Illustrate the instantaneous velocity on the graph of s(t).
(d) Where is the velocity positive? Negative?

Problem 2

Assume that N(t) denotes the size of a population at time t, and that in some conditions, N(t) satisfies

$$N(t) = 2(4^{t/2}).$$

(a) What are $N(1)$, $N(2)$, $N(3)$ and $N(4)$?
(b) What is $\frac{dN}{dt}$? What is the sign of $\frac{dN}{dt}$ and what does it signify?
(c) Is $\frac{dN}{dt}$ constant, increasing or decreasing? What does this tell you about the rate of growth of the population $N(t)$.

Problem 3

Find the derivatives of the following functions:

1. $f(x) = x^2 sin\frac{\pi}{3} + tan\frac{\pi}{4}$
2. $g(x) = 3(x)^{\frac{1}{3}} + x + 4$
3. $g(N) = 4N^2\left(1 - \frac{N}{3}\right)$
4. $N(t) = 5^{t+2}$.