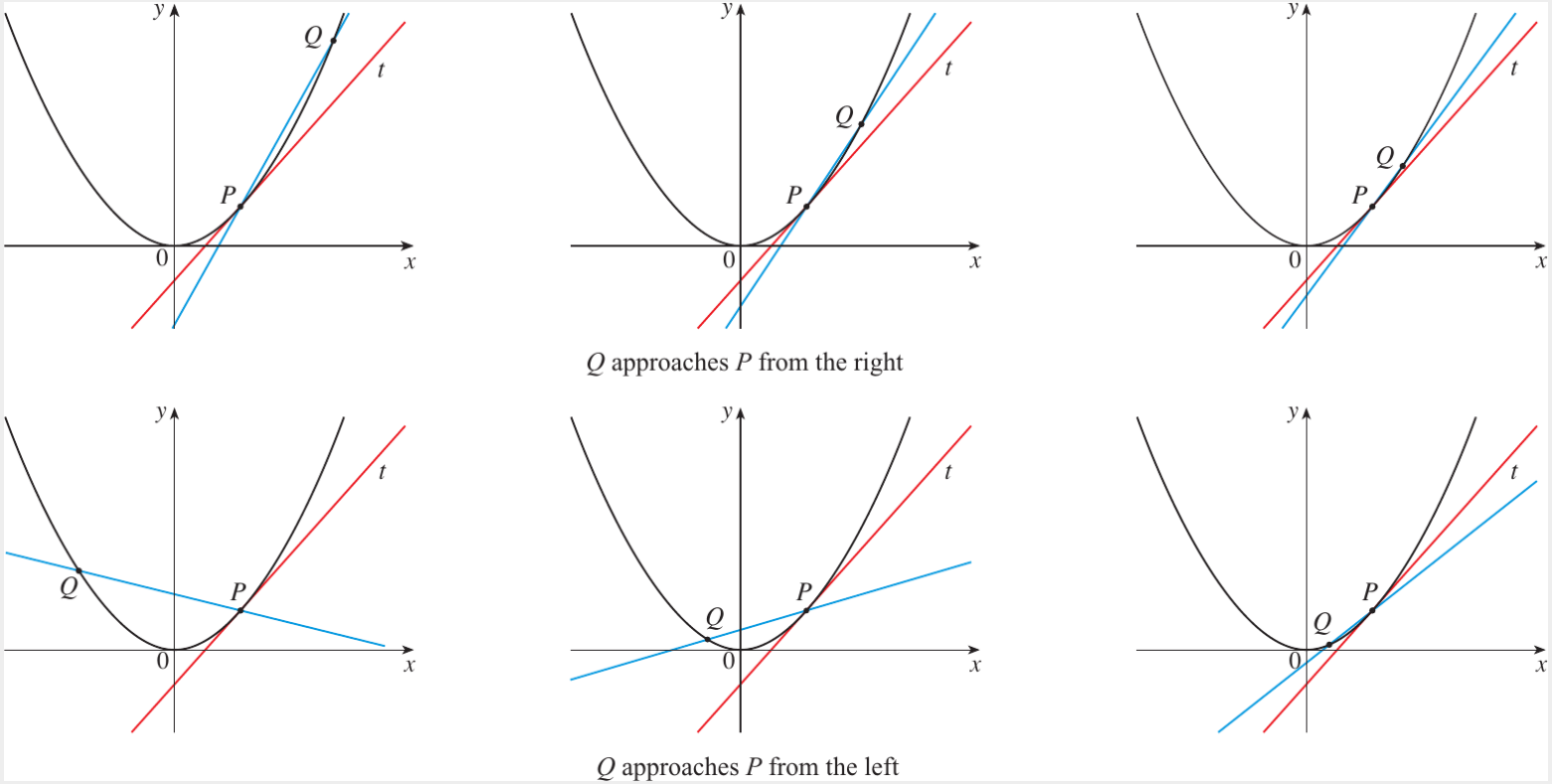
Question of the Day

Why are they called the square numbers?
What about the cube numbers?
Are there other shapes?

On the Docket

Tangent lines.
Limit definition of the derivative.
Worksheet problems on the above.
Concept check in.

Tangent Lines

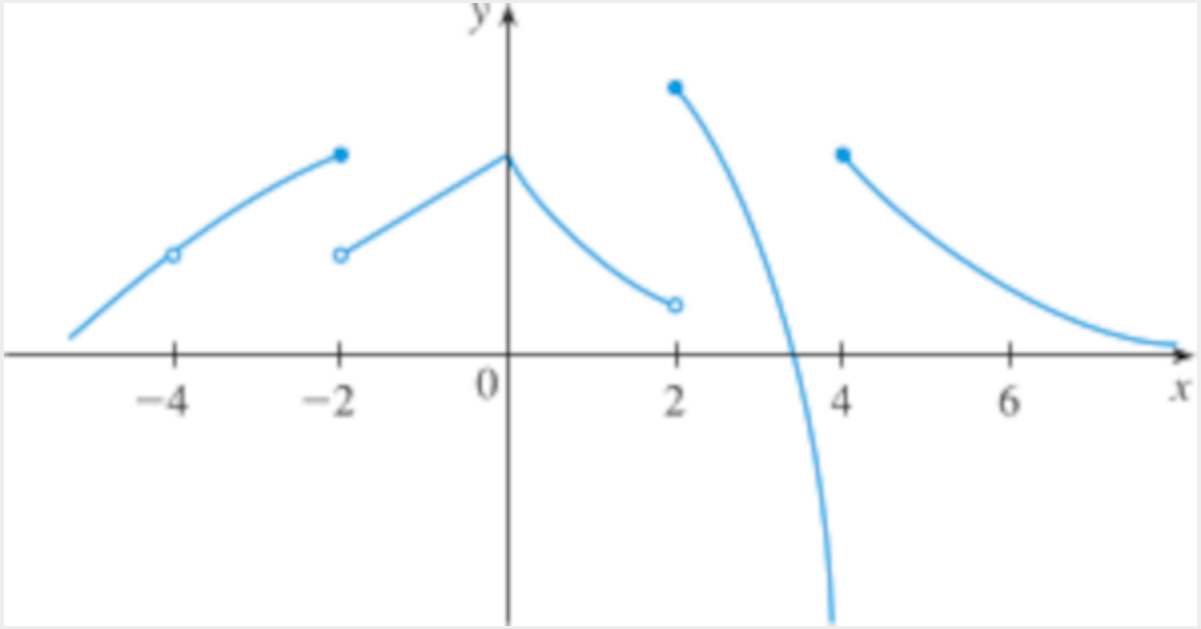


In your own words:

- What is the red line?
- What is the blue line?
- How would you describe their behavior?

Tangent Lines

Consider the following graph (again).



Can you draw the tangent line at:

- $x = -4$
- $x = 0$
- $x = 2$
- $x = 6$

Mathematical Vocabulary

Given a function f, the derivative at a point a is given as

Limit Definition of the Derivative

$$f'(x) = \frac{dy}{dx} = \lim_{a \to x} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- What is the "general idea" of the derivative?
- What is $f(x)$?
- What is $f(a)$?
- What is $f(a+h)$?
- Are these statements equivalent?
- Plug in $x = a$ or $h = 0$. What happens to the expressions?

Introduction to Problem

In climate science, one is interested in understanding the affects of climate change on various species which depend on specific environments to thrive. One of these species is salmon, which are important to the balance of the ecosystems they exist in. Some important measures of a population's health is its overall size, birth rates and death rates.
In this experiment, scientists have studied population dynamics of salmon related to birth rate, specifi- cally, how do rising temperatures affect salmon spawning and the time of hatching. Note that hatching refers to when a baby salmon (a fry) emerges from its egg.

Problem 1

The following data (Sparks et al., 2019) is sent to you, and you are told that it contains measurements of the mean water temperature T in degrees Celsius ($\circ$C) of the spawning location over the hatching period and the number of days H for eggs to hatch after laid by salmon:

Temperature ($^\circ C$)	1.53	2.82	3.46	3.88	4.24	4.97	5.17	5.67
Days until Hatch	212	147	141	133	126	113	110	103
Temperature ($^\circ C$)	6.13	6.46	6.76	7.30	8.47	9.54	9.99	10.04
Days until Hatch	100	94	92	86	77	70	67	67

Problem 1

Temperature ($^\circ C$)	1.53	2.82	3.46	3.88	4.24	4.97	5.17	5.67
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Days until Hatch	100	94	92	86	77	70	67	67

In this problem, we are interested in determining the relationship between the temperature and days.

(a) Write a sentence describing the data in the first column of the top table in the context of the experiment.

(b) Does it make more sense for temperature or days to be the independent variable? Provide justification as to why you chose this variable to be the independent variable.

(c) Using the data tables above, determine the trend of the data. Specifically, compare the columns of the data and answer the following: As the mean temperature rises in the spawning location, what can you conclude about the number of days until hatching?

(d) How could you verify the trend you are observing mathematically?

Problem 2

Temperature ($^\circ C$)	1.53	2.82	3.46	3.88	4.24	4.97	5.17	5.67
Days until Hatch	212	147	141	133	126	113	110	103
Temperature ($^\circ C$)	6.13	6.46	6.76	7.30	8.47	9.54	9.99	10.04
Days until Hatch	100	94	92	86	77	70	67	67

Let H be a function of T.

(a) Compute the average rate of change of H on the interval [1.53, 10.04]. Round your answer to two decimal places (hundredths place).

(b) What are the units of the average rate of change?

(c) Using your answers from (a) and (b), write a sentence describing what the average rate of change on the interval [1.53, 10.04] represents in the context of salmon.

Problem 3

Temperature ($^\circ C$)	1.53	2.82	3.46	3.88	4.24	4.97	5.17	5.67
Days until Hatch	212	147	141	133	126	113	110	103
Temperature ($^\circ C$)	6.13	6.46	6.76	7.30	8.47	9.54	9.99	10.04
Days until Hatch	100	94	92	86	77	70	67	67

Compute the average rate of change on each of the following intervals. You are encouraged to use a calculator in this part of the homework. Round all of your answers to two decimal places (hundredths place).

(a) $[4.97, 7.30]$

(b) $[5.67, 7.30]$

(c) $[6.46, 7.30]$

Problem 4

Temperature ($^\circ C$)	1.53	2.82	3.46	3.88	4.24	4.97	5.17	5.67
Days until Hatch	212	147	141	133	126	113	110	103
Temperature ($^\circ C$)	6.13	6.46	6.76	7.30	8.47	9.54	9.99	10.04
Days until Hatch	100	94	92	86	77	70	67	67

(a) Let T be a real number such that $T \leq 7.30$. Write the difference quotient to find the average rate of change of the interval $[T, 7.30]$. Denote this average rate of change as $A(T)$.

(b) What is $\lim_{T \to 7.30} A(T)$ called?

Problem 5

Temperature ($^\circ C$)	1.53	2.82	3.46	3.88	4.24	4.97	5.17	5.67
Days until Hatch	212	147	141	133	126	113	110	103
Temperature ($^\circ C$)	6.13	6.46	6.76	7.30	8.47	9.54	9.99	10.04
Days until Hatch	100	94	92	86	77	70	67	67

(a) Use your conputations in Question 3 to estimate the instantaneous rate of change at $T = 7.30$.

(b) What are the units of the estimated instantaneous rate of change?

(c) Use your answer in question 5b to write a stentence interpretating the estimated instantaneous rate of change in context.