Question of the Day

Coffe or tea? Both? Neither?
What is your order at the cafe?

Share with a neighbor.

On the Docket

Exponents and logarithms.
Limits of sequences.
Limits of functions.

Recall

For $b > 0,\ b \neq 1$,

$$y = \log_b(x) \quad \iff \quad x = b^y$$

Question: why can't b be 1? Why can't it be negative?

Exponents and Logarithms

Recall

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$$y = \log_b(x) \quad \Longleftrightarrow \quad x = b^y$$

Exercise: simplify the following expressions without a calculator.

(a) $z^{13}\left(\dfrac{3z^2}{z^6}\right)^3$ (b) $\dfrac{(2f)^3}{\sqrt[7]{f^4}}$

(c) $\log_{10}(1000)$ (d) $\log_2(40)$ (e) $\ln\left(\dfrac{3e^3}{\sqrt{7}}\right)$

Problem 1

When the experiment begins, 10 mg of the drug is present in the gut. The passage of the drug from the gut to the blood has first order kinetics; that is, every hour 42% of the drug remaining in the gut is passed from the gut into the blood. No more pills are taken, so no extra drug is added to the gut.

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Closed Form

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After one hour, 58% of the initial amount of the drug remains in the gut. That is, $a1 = 5.8$ mg.
Swarup's friend Abhinav says that, following the pattern, it is obvious that it takes two hours for the gut to have 2.9 mg of the drug in the system, in other words, 29% of the initial amount of the drug. Are they right or are they wrong?

Problem 2

$$a_t = 10 \cdot (0.58)^t$$

(a) Does a_{-5} make sense in the context of the problem? Is there a minimum value for t for which a_t makes sense in this problem? If so, determine the value of a_t for the specific minimum value of n.

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(b) Suppose at time t we know that $a_t = 0.01a_0$. Describe in words the relationship between a_t and a_0.

(c) Calculate the specific time t for which the equation $a_t = 0.01a_0$ is true.

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$$a_t = 10 \cdot (0.58)^t$$

(a) What does $\lim_{t \to \infty} a_t$ mean in the context of this experiment? Without computing, what do you predict its value to be? Justify your answer with physical intuition.

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(b) Compute $\lim_{t\to\infty} a_t$. Justify how you reached your answer. Does this match your prediction?

Problem 3

Hemodialysis is a process by which a machine is used to filter urea and other waste products from an individual's blood when the kidney's fail. Let $c(t)$ be the concentration of urea in the blood after t minutes. We can model the concentration of urea as an exponential decay $c(t) = c_0 e^{-\frac{Kt}{V}}$ where

- K is the mass transfer coefficient measure in ml/min.
- V is the blood volume.
- c_0 is the initial concentration of urea.

Suppose that a patient has a mass transfer value of $K = 340$ ml/min and a blood volume of $V = 32,941$. How long should this patient be put on dialysis to reduce the concentration of urea from 1.65 mg/ml to 0.6 mg/ml?

Problem 4

Banana Slugs

The population of banana slugs of UCSC after t years is modeled using the formula

$$p(t) = \frac{at}{b+t}$$

where a and b are constants. It has been observed that after 2 years there are 1000 slugs on campus. Professor Weissman computed that $\lim_{t\to\infty} p(t) = 2200$.

(a) Use this data to find the constant a.

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(a) Use this data to find the constant a.

(b) Now find the constant b.

Sequences

Does the squence converge? If so, what does it converge to?

(a) $a_n = \dfrac{5}{3^n}$ (b) $a_n = \dfrac{n^3 - 1}{n}$ (c) $a_n = \dfrac{10^n}{1 + 9^n}$

Functions

Evaluate the limit.

(a) $\lim\limits_{n \to \infty} \dfrac{1}{2x + 3}$ (b) $\lim\limits_{n \to -\infty} 0.6^{-t}$ (c) $\lim\limits_{n \to \infty} \dfrac{(2x^2 + 1)^2}{(x - 1)^2(x^2 + x)}$